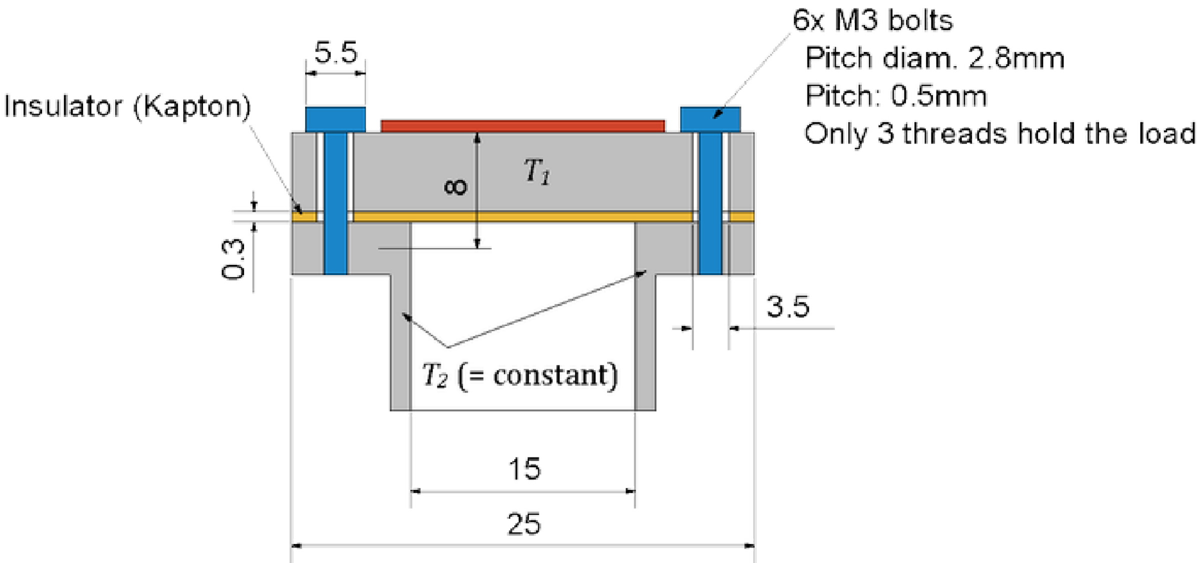
Exercise 4.3 - Thermal Conductivity

v.01

Problem Statement

A heated aluminum flange is fixed to a constant temperature aluminum structure with 6 M3x8 stainless steel bolts. A 0.3 mm thickness Kapton insulator sheet thermally disconnect the two parts.



Numerical data

Thermal conductivities:
Stainless steel: 16 W/(m·K)
Kapton: 0.12 W/(m·K)
Thermal conductance of the contact: 10^7 W/(m ^{2}·K) Note: Kapton-metal and metal-metal are considered to be identical
Temperatures of the structures [°C]:

In[ ]:=
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T₁ = 70;
T₂ = 55;
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Hint: the bolts have a simplified geometry, i.e. the threads have a perfectly rectangular shape, with a contact on the whole length.

Questions

- What is the thermal power transferred from the flange to the structure?
- Which part has the highest contribution: the bolts or the direct conduction through the structure?

Solution

Number of bolts:

In[•]:= $N_b = 6;$

Contact of the flange

Inner diameter [m]:

In[•]:= $d_{in} = 15 \times 10^{-3};$

Outer diameter [m]:

In[•]:= $d_{out} = 25 \times 10^{-3}$

Out[•]=

$$\frac{1}{40}$$

Surface of contact [m ^{2}]:

In[•]:= $A_{flange} = \frac{\pi \left(d_{out}^2 - d_{in}^2\right)}{4};$

M3 bolts holding on 3 threads, stainless steel

Length of the bolt (head and thread) [m]:

In[•]:= $L = 8 \times 10^{-3};$

Diameter of contact of the bolt head [m]:

In[•]:= $d_1 = 5.5 \times 10^{-3};$

Mean diameter of the thread (pitch diameter) [m]:

In[•]:= $d_2 = 2.8 \times 10^{-3};$

Bore diameter of the bolt [m]:

In[•]:= $d_3 = 3.5 \times 10^{-3};$

Pitch of the bolt [m]:

In[•]:= $p = 0.5 \times 10^{-3};$

Thread angle [rad]:

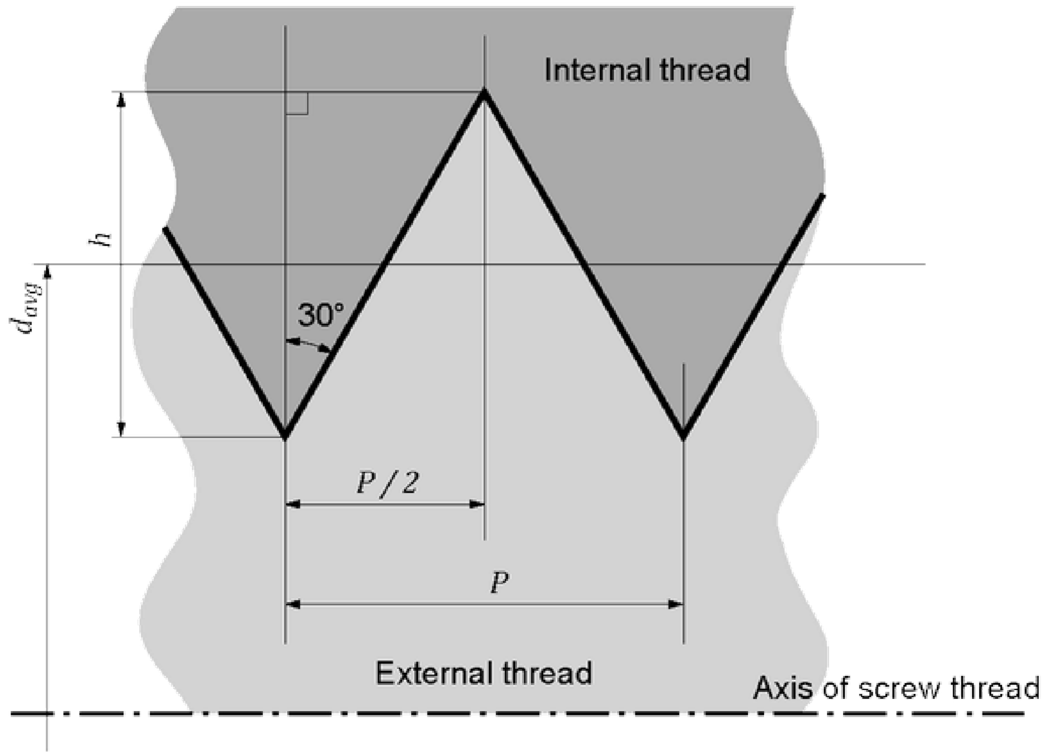
In[∘]:= $\theta = 30\,\frac{2\,\pi}{360};$

Surface of contact of the bolt (3 threads) [m ^{2}]:

In[∘]:= $A_b = \frac{3\,\pi}{2\,\mathrm{Sin}[\theta]}\,d_2\,p$

Out[∘]=

0.0000131947



Surface of a ring: $A_{ring} = \pi \cdot h \cdot d_{avg}$

and $h = \frac{P/2}{\sin(30°)}$

Surface of contact under the head [m ^{2}]:

In[∘]:= $A_{head} = \frac{\pi}{4}\,\left(d_1^2 - d_3^2\right)$

Out[∘]=

0.0000141372

Kapton sheet

Thickness [m]:

In[∘]:= $h_K = 0.3\times10^{-3};$

Thermal parameters

Thermal conductivities of stainless steel and Kapton [W/(m·K)]:

In[∘]:= $\lambda_s = 16;$
$\lambda_K = 0.12;$

Thermal contact conductance of the Kapton-Aluminum interface [W /(m ^{2}· K)]:

In[∘]:= $\varkappa_K = 10^7;$

Heat transfer

Heat transfer at Aluminum interface [W]:

In[∘]:= $$Q_f[h\_] = \left(\frac{2}{\varkappa_K A_{flange}} + \frac{h}{\lambda_K A_{flange}}\right)^{-1} (T_1 - T_2);$$

Heat transfer by the bolts [W]:

In[∘]:= $$Q_b = N_b \left(\frac{1}{\varkappa_K}\left(\frac{1}{A_b} + \frac{1}{A_{head}}\right) + \frac{L}{\lambda_s \frac{\pi d_2^2}{4}}\right)^{-1} (T_1 - T_2)$$

Out[∘]=

1.10815

Heat transfer of the system [W]:

In[∘]:= $Q[h\_] = Q_f[h] + Q_b;$

Ratio between the heat transferred through the bolts only and through the whole system:

In[∘]:= $$\rho_{tot}[h\_] = \frac{Q_b}{Q_f[h] + Q_b};$$

In[∘]:= $Q[h_K]$

Out[∘]=

2.99296

In[∘]:= $\rho_{tot}[h_K]$

Out[∘]=

0.370254

This means that 37% of the heat is transferred through the bolts.

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In[∘]:= Plot[Q[h], {h, 1 × 10^-6, 0.5 × 10^-3}, AxesLabel → {HoldForm[h in m], HoldForm[Q in W]},
        tracé                                    étiquette des axes  maintiens forme       maintiens forme
          PlotLabel → None, LabelStyle → {GrayLevel[0]}]
          étiquette de tr···  aucun  style d'étiquette  niveau de gris
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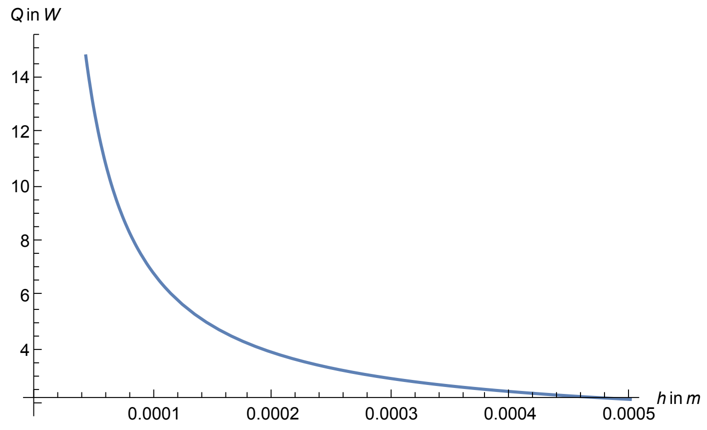
Out[∘]=

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In[∘]:= Plot[ρtot[h], {h, 1 × 10^-6, 0.5 × 10^-3}, AxesLabel → {HoldForm[h in m], HoldForm[rho]},
        tracé                                       étiquette des axes  maintiens forme       maintiens forme
          PlotLabel → None, LabelStyle → {GrayLevel[0]}]
          étiquette de tr···  aucun  style d'étiquette  niveau de gris
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Out[∘]=